

## Inquisitive Semantics: Conditionals, Questions, and Disjunction

I present an *inquisitive semantics* for a language of propositional logic in update format. Next to a standard notion of entailment, the semantics comes with a logical notion of *licensing*, a strict notion of contextual relatedness. I show that the interpretation assigned to disjunction meets Grice's opinion on its standard use. I illustrate the semantics with some examples which involve the interplay of conditionals, questions and disjunction.

The **syntax** of the logical language is standard. The vocabulary consist of a set of propositional variables  $\wp$  and the *falsum*  $\perp$ ; the basic operators are  $\rightarrow, \vee, \wedge$ . We add:

$$\neg\varphi =_{def} (\varphi \rightarrow \perp) \quad \top =_{def} \neg\perp \quad !\varphi =_{def} \neg\neg\varphi \quad ?\varphi =_{def} (\varphi \vee \neg\varphi)$$

We will interpret sentences as update functions on states..

### Indices, States, and Possibilities in States

The set of *indices* is the set of functions  $i$ , such that for all  $p \in \wp$ :  $i(p) \in \{0, 1\}$

A *state*  $\sigma$  is a reflexive and symmetric relation on a subset of the set of indices

A *possibility* in a state  $\sigma$  is a largest set of indices  $\rho$  such that for all  $i, j \in \rho$ :  $\langle i, j \rangle \in \sigma$

**Indifferentiation** The *indifferentiation* of  $\sigma$ ,  $\sigma^* = \{\langle i, j \rangle \mid \langle i, i \rangle \in \sigma \ \& \ \langle j, j \rangle \in \sigma\}$

When  $\sigma \neq \sigma^*$ , i.e., there are  $\langle i, i \rangle, \langle j, j \rangle \in \sigma$  such that  $\langle i, j \rangle \notin \sigma$ , there is an **issue** in  $\sigma$ .

If there is an issue in  $\sigma$ , we find more than one possibility in  $\sigma$ .

Two possibilities in  $\sigma$  may overlap, the set of possibilities in  $\sigma$  need not form a partition.

### Inquisitive Update Semantics

$$\sigma[p] = \{\langle i, j \rangle \in \sigma \mid i(p) = 1 \ \& \ j(p) = 1\}, \text{ for all } p \in \wp$$

$$\sigma[\perp] = \emptyset$$

$$\sigma[\varphi \rightarrow \psi] = \{\langle i, j \rangle \in \sigma \mid \text{for all } \iota \in \{i, j\}^2: \text{ if } \iota \in \sigma[\varphi], \text{ then } \iota \in \sigma[\varphi][\psi]\}$$

$$\sigma[\varphi \vee \psi] = \sigma[\varphi] \cup \sigma[\psi]$$

$$\sigma[\varphi \wedge \psi] = \sigma[\varphi][\psi]$$

### Inquisitiveness and Informativeness

$\varphi$  is *inquisitive in*  $\sigma$  iff  $\exists \langle i, j \rangle \in \sigma$ :  $\langle i, i \rangle \in \sigma[\varphi] \ \& \ \langle j, j \rangle \in \sigma[\varphi]$ , but  $\langle i, j \rangle \notin \sigma[\varphi]$

$\varphi$  is *informative in*  $\sigma$  iff  $\exists \langle i, i \rangle \in \sigma$ :  $\langle i, i \rangle \in \sigma[\varphi]$ , and  $\exists \langle j, j \rangle \in \sigma$ :  $\langle j, j \rangle \notin \sigma[\varphi]$

$\varphi$  is *inquisitive/informative* iff for some state  $\sigma$ :  $\varphi$  is inquisitive/informative in  $\sigma$

The syntax does *not* distinguish between indicative and interrogative sentences, but we can distinguish three *semantic categories* of sentences.

### Questions, Assertions and Hybrids

$\varphi$  is a *question* iff  $\varphi$  is not informative     $\varphi$  is an *assertion* iff  $\varphi$  is not inquisitive

$\varphi$  is a *hybrid* iff  $\varphi$  is neither a question nor an assertion

### Entailment and Licensing

$\sigma$  *supports*  $\psi$  iff  $\sigma[\psi] = \sigma$

$\sigma$  *licenses*  $\psi$  iff every possibility in  $\sigma^*[\psi]$  is the union of a set of possibilities in  $\sigma$

$\varphi$  *entails/licenses*  $\psi$  iff for every state  $\sigma$ :  $\sigma[\varphi]$  supports/licenses  $\psi$

## Grice on Disjunction

A standard (if not *the* standard) employment of “or” is in the specification of possibilities (one of which is supposed by the speaker to be realized, although he does not know which one), each of which is relevant in the same way to a given topic. [Grice (1989), p. 68]

## Inquisitive Semantics Meets Grice

A disjunction like  $p \vee q$  is a hybrid. It is both informative and inquisitive. It gives rise to two possibilities. A disjunction  $p \vee q$  is licensed in a state  $\sigma$  iff each of the two possibilities equals a union of possibilities in  $\sigma$ , i.e., each of them is strictly related to the issue in  $\sigma$ .

## Illustration: Conditionals, Questions, and Disjunction

Consider sentence (1).

- (1) If Alf goes, will Bea go as well?  $p \rightarrow ?q$

The semantics characterizes (1) as a question. Up to equivalence, the only two informative sentences licensed by (1) are the two assertions in (2).

- (2) a. (Yes.) If Alf goes, then Bea will go as well.  $p \rightarrow q$   
b. (No.) If Alf goes, then Bea will not go.  $p \rightarrow \neg q$

Both assertions in (2) also *entail* the question in (1). That they have in common with  $\neg p$ . The difference is that the negation of the antecedent of (1) is not licensed by (1).

The semantics predicts that the sentences in (2) are also natural responses to (3) and (4).

- (3) If Alf goes, Bea will go as well, or Bea will not go.  $p \rightarrow (q \vee \neg q)$   
(4) If Alf goes, Bea will go as well, or if Alf goes, Bea will not go.  $(p \rightarrow q) \vee (p \rightarrow \neg q)$

Standard semantics rather predicts, that (5) is the appropriate reaction:

- (5) (Of course!) If Alf goes, either Bea goes as well, or she does not go!  $p \rightarrow !(q \vee \neg q)$   
Alternative translation, like the other one, equivalent with  $\top$   $!(p \rightarrow (q \vee \neg q))$

The exclamation mark in (5) forces an intonation pattern which gives it an assertive force. The more neutral presentations of the sentences in (3) and (4) allow for several other intonation patterns, which give rise to an inquisitive interpretation with the same conversational effect as the conditional question in (1).

As our final example, consider the conditional question in (6).

- (6) If Alf goes, or Cor goes, will Bea go as well?  $(p \vee q) \rightarrow ?r$

Not just (7a) and (b), but also (c) and (d) are possibilities that (6) gives rise to.

- (7) a. If Alf or Cor goes, then Bea goes as well.  $(p \vee q) \rightarrow r$   
b. If Alf or Cor goes, then Bea does not go.  $(p \vee q) \rightarrow \neg r$   
c. If Alf goes, Bea does go, and if Cor goes, Bea doesn't go.  $(p \rightarrow r) \wedge (q \rightarrow \neg r)$   
d. If Alf goes, Bea doesn't go, and if Cor goes, Bea does go.  $(p \rightarrow \neg r) \wedge (q \rightarrow r)$

With the assertive closure of the antecedent, as in  $!(p \vee q) \rightarrow ?r$ , only (a) and (b) count.

**References** Grice (1989) *Studies in the Ways of Words*. Groenendijk (1999) The Logic of Interrogation *SALT 9*. Isaacs & Rawlins (2007) Conditional Questions. Nelken & Francez (2001) Bilattices and the Semantics of Natural Language Questions *L&P*. Rooth (1985) *Association with Focus* PhD thesis UMass. Simons (2000) *Issues in the Semantics and Pragmatics of Disjunction* Garland NY. Simons (2001) Disjunctions and Alternatives *L&P*. Velissaratou (2000) *Conditional Questions and Which-Interrogatives* MA thesis UAmsterdam.